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14. ABSTRACT
New direct and analytically preconditioned frequency and time domain integral equation solvers have been developed. First, mechanisms for compressing scattering matrices in static and relatively low-frequency environments will be pursued, and used in the construction of direct frequency domain integral equation solvers applicable to objects roughly 40 wavelengths in size. Second, hierarchical and Calderon preconditioned time-domain integral equation solvers were developed. These solvers remain repidly convergent and stable even when applied to very low-frequency problems and/or very densely meshed structures. Third, the methods developed gave rise to new techniques for designing numerical quadratures and for computing singular value decompositions.

15. SUBJECT TERMS
Integral equations, fast multipole method, fast direct solvers, fast analytically preconditioned solvers

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1. PROGRAM GOALS AND OVERVIEW

Today, Laplace, Helmholtz, and wave equation FMM-accelerated static-, frequency-, and time-domain integral equation solvers permit the *analysis* of electromagnetic interactions with structures of geometric complexity and dimensions inconceivable just a decade ago. Unfortunately, many FMM success stories notwithstanding, recently few radically new FMM algorithmic constructs have been proposed. Instead, FMM research gradually has evolved into a hunt for application-specific improvements with limited range. Perhaps not surprisingly, FMM technology has failed to penetrate new fields and within the electromagnetic engineering realm, its impact on ultra-large scale analysis, design and optimization, and inverse scattering remains low.

To reverse this stagnation in fundamental FMM research and to initiate the development of new FMM-like technologies in fields that hitherto have resisted FMM application, we studied a pair of fundamental existence questions:

I. *Existence and construction of compressed scattering matrix representations*: Are $N-$port (N by N) electromagnetic scattering matrices redundant? That is, do compressed representations of such matrices involving $O(N^{\alpha} \log^{\beta} N)$ data with small α and β exist?

II. *Existence and construction of fast direct solvers*: If so, are these compressed representations amenable to fast multiplication by a vector? That is, can they be used to construct fast direct (as opposed to iterative) electromagnetic solvers? Alternatively, do there exist analytical preconditioners that turn iterative solvers into quasi-direct ones?

First, mechanisms for compressing scattering matrices in static and relatively low-frequency environments were pursued, and used in the construction of direct solvers for analyzing like scenarios. These algorithms exploited the low-rank nature of scattering matrices of electromagnetically small objects. The CPU time requirements of these schemes scales as $N \log^2(N)$, with N the number of scatterer features; they are quite efficient for scatterers up to approximately 40 wavelengths across. Second, new analytical preconditioners for regularizing time-domain integral equations were pursued. These resulting schemes exploited new time-domain Calderon identities and were shown to dramatically reduce operation counts of present time domain integral equation solvers.

2. PROGRAM ACHIEVEMENTS

Our work under this contract can be roughly subdivided into four parts: Fast algorithms in the frequency domain, fast algorithms in the time domain, numerical tools for fast algorithms, and collateral issues. Following is a discussion of the issues addressed by our effort.

1. We designed and implemented a fast direct solver for objects in both two and three dimensions that are long and thin. The codes include both the ``fast" element of the algorithm and the rapidly convergent discretizations in various situations. An important feature of the scheme is its effectiveness in both the high and low-frequency environments. This work has been reported in [1]; the scheme has been generalized to two dimensions, the code developed, and a paper reporting these results is in preparation.

2. One of spin-offs of this activity is an observation that given a set of n very general bounded functions $\varphi_1, \varphi_2, ..., \varphi_n$ (with a finite n), there exist stable n-point interpolation and quadrature formulae exact on the linear space spanned by the functions $\varphi_1, \varphi_2, ..., \varphi_n$. This is a somewhat unexpected observation, and its proof depends entirely on linear algebraic arguments; we have also implemented numerical algorithms for the construction of such interpolation and quadrature formulae applicable in many situations of practical interest (in addition to the uses of such schemes in numerical scattering theory). These results are reported in [2].

3. One of the principal tools in the development of algorithms of this type is the concept of ``skeletonization" (see, for example, [6]). Recently, it turned out that under certain conditions (highly relevant to the design of scattering algorithms), introducing a randomized element into skeletonization schemes leads to radically improved efficiency *and* reliability. More specifically, given a linear operator $A: R^n \to R^m$, the algorithm reported in [3] constructs an approximate Singular Value Decomposition (SVD) of A for a cost of the order (Q+Q*). log (k/ε), where Q is the cost of applying A to an arbitrary vector, Q* is the cost of applying A* to an arbitrary vector, and ε is the accuracy to which the SVD is to be calculated. It should be observed that this estimate is extremely favorable whenever there exists a ``fast" algorithm for the application of the matrices A and A* to arbitrary vectors, and of much less use when the operator to be compressed is specified as a dense matrix (in this case, the procedure reduces the memory footprint of the algorithm, but does not change its asymptotic CPU time requirements. These results are reported in [3].

4. More recently, a scheme was constructed using randomization to accelerate the construction of the SVD when the operator to be compressed is represented as a dense matrix. Given a *dense* matrix A of dimensionality n x m and approximate rank k, it produces the SVD of A for a cost proportional to $m.n.\log((k+1/\varepsilon)$, which is a new complexity result for the construction of the SVD of a matrix of limited rank. These results are reported in [4].

5. A novel regularized combined field integral equation pertinent to the analysis of scattering from 2-D perfect electrically conducting objects is presented IN [7]. The equation is immune from ill-conditioning due to dense spatial discretizations and appears free from internal resonances. The regularization is achieved via analytical inversion of the hypersingular part of the combined field integral equation, without negative repercussions on computational complexity. Numerical results are presented that show the effectiveness of the proposed formulation.

6. A new technique for preconditioning electric field integral equations by leveraging Calderon identities is presented [8]. In contrast to all previous Calderon preconditioners, the proposed preconditioner is purely multiplicative in nature and applicable to open and closed

structures. The preconditioned equation is obtained by multiplying the EFIE matrix (obtained with standard codes) times properly constructed matrices. These additional matrices can be obtained easily and do not require further use of numerical integration. In other words the preconditioner presented can be immediately integrated into a pre-existing code maintaining the effectiveness of Calderòn methods. The multiplicative preconditioner is obtained by exploring the relationship between ordinary and barycentric Rao-Wilton-Glisson basis functions, obtaining a set of analytic coefficients that allow the mapping from one basis set to the other. The matrices required by the preconditioner in addition to the usual MoM matrix are highly sparse and containing O(N) elements. As a consequence the new preconditioner has a minimal impact on the overall computational cost.

7. A new Time-Domain Electric Field Integral Equation (TD-EFIE) is presented [9]. Leveraging the Calderon identities, this new equation can be shown to be well-conditioned. In particular it does not suffer from the dense grid breakdown unlike the classic TD-EFIE. The discretization strategy for this preconditioned TD-EFIE was analyzed in great detail. The Buffa-Christiansen basis functions used in this discretization are introduced. Finally, numerical examples are presented to show both the performance and accuracy of the proposed scheme.

8. Novel time domain integral equations for simulating scattering from perfect electrically conducting objects are presented in [10]. They are free from DC and resonant instabilities plaguing the standard electric field integral equation. The new equations are obtained using operator manipulations originating from the Calderon identities. The theoretical motivations behind the definition of the new equations are explored in detail and numerical results concerning their theoretically predicted behavior are presented.

9. A new hierarchical approach to discretize the time domain electric field integral equation is presented in [11]. The proposed technique is somewhat limited in scope, as it requires dyadic mesh refinement, but nonetheless is very effective for a large class of structures. A hierarchical current expansion is used in a method of moments marching on in time scheme. The formulation gives rise to well conditioned and fast converging linear systems and it is immune from the time domain low frequency breakdown. Numerical results are presented that show the applicability and the effectiveness of the proposed formulation and its performance is compared with those attainable with existing technologies.

10. A hierarchical approach to discretize the time domain electric field integral equation is presented [12]. The proposed technique can be applied to arbitrary meshes and gives rise to well-conditioned marching on in time linear systems immune to low-frequency breakdown. Numerical results that demonstrate the effectiveness of the proposed technique in comparison with existing methods are presented.

4. PERSONNEL AND INFRASTRUCTURE SUPPORTED

4.1 Faculty

1. Eric Michielssen, Center for Computational Electromagnetics, Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign (until August 2005) and Radiation Laboratory, University of Michigan (until present).
2. Vladimir Rokhlin, Departments of Computer Science and Mathematics, Yale University.

4.2 Postdocs and Students

Students listed below were supported, entirely or in part (salary and/or infra-structure), by the DARPA-DSO/AFOSR grant. (UIUC = University of Illinois at Urbana-Champaign, UM = University of Michigan, YALE = Yale University).

1. Ali Yilmaz, Ph.D. and postdoc, UIUC.
2. Hakan Bagci, Ph.D., UIUC, and postdoc UM, August 2006.
3. Francesco Andriulli, Ph.D., UM, expected graduation December 2007
4. Mark Tygert, Ph.D and postdoc, YALE
5. Gunnar Martinsson, postdoc, YALE

5. CONTINUING APPLICATIONS - TECHNOLOGY TRANSITIONS

Computational technologies developed under this grant are continuing to be applied to electromagnetic compatibility/interference (EMC/EMI) and high power microwave (HPM) problems of interest to the DoD/AF. Specifically, they are being used in a phase II SBIR grant (AFRL Kirtland, program manager Joe Yakura) aimed at producing a user friendly, time domain integral equation code for analyzing the penetration of wideband electromagnetic pulses into enclosures containing sensitive electronics (e.g., a personal computer containing multiple circuit boards or a missile cone containing a nonlinear amplifier) and cable-loaded airframes containing such systems. Given the broadband nature of the electromagnetic threats to these often nonlinear systems, the fast time domain integral equation solvers developed under this grant have proven instrumental in tackling these problems. Our methodologies now are recognized as one of very few possible avenues for analyzing the above phenomena using numerically rigorous tools.

With funding from AFOSR MURI grant "Electromagnetics of Antennas and Arrays Designed Using Novel Electronic Materials and Conformal to Large Complex Bodies", (managed by Dr. Arje Nachman, AFOSR), computational technologies developed under this grant are being applied to the study of complex and potentially nonlinear antenna feed networks. This effort, has demonstrated the applicability of computational technologies developed under this program to the time-domain analysis of broadband, airframe installed, log-periodic monopole antennas fed by complex cable networks. Current efforts focus on applying the same technology to the characterization of nonlinearly loaded microstrip antennas.

Other grants that leverage computational technologies developed under this program include:

1. An SRC for time-domain modeling of on-chip interconnects modeling of chip interconnects.
2. A Boeing grant for frequency-domain modeling of scattering from wedges and the

development of applicable generalized impedance boundary conditions.

6. PUBLICATIONS

Journal Papers

Papers listed below not readily available in the archival literature are provided in Appendix.

[1] P.G. Martinsson and V. Rokhlin, "A fast direct solver for scattering problems involving elongated structures," Journal of Computational Physics, Volume 221, Issue No. 1, 2007, Pages 288-302.

[2] P.G. Martinsson, V. Rokhlin, and M. Tygert, "On interpolation and integration in finite-dimensional spaces of bounded functions," Communications in Applied Mathematics and Computational Science (CAMCoS), V. 1, pp. 133-142, 2006.

[3] P.G. Martinsson, V. Rokhlin, and M. Tygert, "A randomized algorithm for the approximation of linear operators," YALEU/DCS/RR-1352, 2006.

[4] F. Woolfe, E. Liberty, V. Rokhlin, M. Tygert, "A fast randomized algorithm for the approximation of matrices - preliminary report," YALEU/DCS/RR-1380, 2007.

[5] V. Rokhlin and M. Tygert, "Fast algorithms for spherical harmonic expansions," SIAM J. Sci. Comput. , 27(6): 1903-1928, 2006.

[5] P.G. Martinsson and V. Rokhlin,``A Fast Direct Solver for Boundary Integral Equations in Two Dimensions", Journal of Computational Physics, 205 (2005), pp. 1-23.

[6] P.G. Martinsson and V. Rokhlin, "An Accelerated Kernel-Independent Fast Multipole Method in One Dimension", SIAM J. Sci. Comput. , 29(3): 1160-1178 (2007).

[7] F. Andriulli, E. Michielssen, *A Regularized Combined Field Integral Equation for Scattering from 2D Perfect Electrically Conducting Objects* , IEEE Transactions on Antennas and Propagation, Vol. 55 n.9, Sep. 2007, pp. 2522–2529.

[8] F. Andriulli, K. Cools, H. Bagci, F. Olyslager, S. Christiansen, A. Buffa, E. Michielssen, *A Multiplicative Calderon Preconditioner for the Electric Field Integral Equation.* University of Michigan Radiation Lab technical report, radEM-003

[9] K. Cools, F. Andriulli, E. Michielssen, *On the Calderon Techniques in Time Domain part I: Conditioning*. University of Michigan Radiation Lab technical report, radEM-004

[10] F. Andriulli, K. Cools, E. Michielssen, *On the Calderon Techniques in Time Domain part II: Stability*. University of Michigan Radiation Lab technical report, radEM-005

[11] F. Andriulli, H. Bagci, F. Vipiana, G. Vecchi, E. Michielssen, *A Marching-on-in-Time Hierarchical Scheme for the Time Domain Electric Field Integral Equation*. accepted for publication to appear on IEEE Transactions on Antennas and Propagation.

[12] F. Andriulli, H. Bagci, F. Vipiana, G. Vecchi, E. Michielssen, *Hierarchical Regularization of the Time Domain Electric Field Integral Equation*. University of Michigan Radiation Lab technical report, radEM-006

Selected Refereed Conference Papers

[13] A. E. Yılmaz, J. M. Jin, and E. Michielssen, "A dual/variable time stepping framework for TDIE-based hybrid field-circuit simulators," in *Proc. USNC/URSI Rad. Sci. Meet.*, July 2005.

[14] A. E. Yılmaz, J. M. Jin, and E. Michielssen, "Hybrid time-domain integral equation/circuit solvers for nonlinearly loaded antennas on complex platforms," in *Proc. USNC/URSI Rad. Sci. Meet.*, July 2005.

[15] H. Bağcı, A. E. Yılmaz, and E. Michielssen, "Full TDIE-based modeling of electromagnetic coupling into lossy multiconductor cables on electrically large platforms," in *Proc. USNC/URSI Rad. Sci. Meet.*, July 2005.

[16] H. Bagci, A. E. Yilmaz, and E. Michielssen, "A fast hybrid TDIE-FDTD-MNA scheme for analyzing cable-induced transient coupling into shielding enclosures," Proceedings of the 2005 IEEE International Symposium on Electromagnetic Compatibility, pp. 828-833 Vol. 3, Chicago, IL, USA, 2005.

[17] V. Bagci, A. E. Yilmaz, and E. Michielssen, "EMC/EMI analysis of electrically large and multiscale structures loaded with coaxial cables by a hybrid TDIE-FDTD-MNA approach," Proceedings of the 2005 IEEE Antennas and Propagation Society International Symposium, pp 14-17, vol. 2B, Washington, DC, USA, 2005.

[18] P. L. Jiang and E. Michielssen, "Multilevel plane wave time domain-enhanced MOT solver for analyzing electromagnetic scattering from objects residing in lossy media," Proceedings of the 2005 IEEE Antennas and Propagation Society International Symposium, pp. 447-50, vol. 3B, Washington, DC, USA, 2005.

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[22] C. Qin, L. Mingyu, L. Meng, and E. Michielssen, "Time domain integral equation based analysis for thin scatterers using impedance boundary conditions," Proceedings of the 2005 IEEE Antennas and Propagation Society International Symposium, 155-158, vol. 3A, Washington, DC, USA, 2005.

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[25] A. E. Yılmaz, Z. Lou, E. Michielssen, and J. M. Jin "A parallel time-domain adaptive integral method-accelerated single-boundary finite element-boundary integral solver," in *Proc. USNC/URSI Rad. Sci. Meet.*, July 2006, p. 297.

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